Closing tonight: 3.3
Closing next Fri: 3.4(1), 3.4(2)
Exam 1 is Tuesday in normal quiz section. See website for a reminder of rules and a review sheet/study advice.

Entry Task: Use the quotient rule to find the derivatives of

1. $y = \dfrac{\sin(x)}{\cos(x)}$
2. $y = \dfrac{1}{\sin(x)}$

3.3 Trig Derivatives (*continued*)

Last time, we showed

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} = \cos(x)$$

By using a trig identity and the fact

$$\lim_{h\to 0} \frac{\sin(h)}{h} = 1.$$

Thus,

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

Similarly, it can be shown that

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\frac{d}{dx}(\cos(x)) = -\sin(x)$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
$\frac{d}{dx}(\sec(x)) = \sec(\mathrm{x})\tan(\mathrm{x})$	$\frac{d}{dx}(\csc(x)) = -\csc(\mathrm{x})\cot(\mathrm{x})$

What is the derivative of:

1. $y = x^3 \tan(x)$

2. $y = e^x \cos(x) + \frac{3x}{2}$

Side note: You can now use

$$\lim_{h\to 0}\frac{\sin(h)}{h}=1$$

Examples: Evaluate

1. $\lim_{x\to 0}\frac{\sin(3x)}{3x}=$

2. $\lim_{x\to 0}\frac{\sin(5x)}{4x}=$

3. $\lim_{x\to 0}\frac{\sin(6x)-3e^{x}\sin(6x)}{x}=$

3.4 Chain Rule

The **composition** of two function is defined by

$$(f \circ g)(x) = f(g(x))$$

Example:

If $f(x) = \sin(x)$, $g(x) = x^3$, then

$$(f \circ g)(x) = f(g(x)) = \sin(x^3).$$

Chain Rule:

$$\frac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big) g'(x)$$

Also written as: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Example:

$$\frac{d}{dx} \sin(x^3) = \cos(x^3)\, 3x^2$$

Here is a brief "proof sketch" for the chain rule:

From the definition of derivative

$$\begin{aligned}
\frac{d}{dx}f\big(g(x)\big) &= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{h} \\
&= \lim_{h\to 0}\left(\frac{f(g(x+h))-f(g(x))}{h}\;\frac{g(x+h)-g(x)}{g(x+h)-g(x)}\right) \\
&= \lim_{h\to 0}\left(\frac{f(g(x+h))-f(g(x))}{g(x+h)-g(x)}\right)\left(\frac{g(x+h)-g(x)}{h}\right) \\
&= f'\big(g(x)\big)g'(x)
\end{aligned}$$

Examples: Find the derivative

1. $y = (2x^2 + 1)^2$

2. $y = e^{\sin((2x+1)^3)}$

3. $y = \tan(3x + \cos(4x))$

4. $y = \sin^4(x)$

5. $y = \sin(x^4)$

Identify the “first” rule you would use to differentiate these functions:
(sum, product, quotient or chain?)

1. $y = \sqrt{\sin(x) + x^2 + 1}$

2. $y = \frac{x^4}{\sin(5x+1)}$

3. $y = \sqrt[3]{4x + 1}\cos(\sin(2x))$

4. $y = e^{\tan(x)} - 5(x^8 + 1)^{50}$

5. $y = \left(\frac{x^2-1}{x^4+1}\right)^{10}$